

# Chapter 14

## Improving Push-based FRP

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*Category: Research*

**Abstract:** Push-based implementations of Functional Reactive Programming allow for writing reactive programs in a declarative style and execute them efficiently. Previous approaches in this area are not able to combine simultaneously occurring internal events. This may lead to efficiency problems and introduction of inconsistent intermediate states. Most of these implementations also lack declarative means to describe systems with feedbacks. We limit ourselves to the data flow aspects of FRP and present an implementation which does not suffer from these deficiencies. This leads to a foundation for FRP which is more declarative than previous solutions and more efficient at the same time.

### 14.1 INTRODUCTION

Functional Reactive Programming (FRP) makes it possible to implement reactive systems in a declarative way. It is based on the notions of discrete and continuous signals. A discrete signal is a sequence of discrete points in time with a value attached to each of it. A continuous signal is a time-dependent value. FRP systems provide means to construct new signals from existing ones in purely declarative ways. Especially, they support switching, that is, the creation of signals whose behaviors switch to the behaviors of arbitrary other signals.

There are two principal approaches to implement FRP:

**Push-based implementations** react to external events by updating their internal state and influencing the environment.

**Pull-based implementations** regularly poll interesting parts of the environment's state and modify their internal state and the environment accordingly.

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Typically, push-based implementations recalculate only those parts of the internal state and the environment which need updating. This is an advantage over pull-based approaches which always recalculate everything.

However, previous push-based systems fell short of providing the declarativeness and flexibility pull-based systems offer. A common problem is *missing treatment of simultaneity*. When discrete signals are combined, values belonging to the same time are not identified as simultaneously occurring. This can result in multiple reactions of the system instead of one large reaction which can reduce performance and introduce inconsistent intermediate states. In addition, most push-based FRP systems do not allow *declarative descriptions of feedbacks*. Feedbacks are dependencies of a subsystem's input on its own output.

In this paper we attack these two problems. We discuss an FRP foundation which is essentially a data flow system. This means that, in contrast to a full FRP system, it lacks support for continuous signals and switching. We introduce our system by example and give a discrete signal semantics in section 14.2. Afterwards we present a push-based implementation of our approach. We make thus the following contributions:

- In section 14.3, we implement a framework for describing systems of reactive components communicating via signals. Our implementation accumulates parts of a system's initialization action for later execution. This allows declarative descriptions of feedbacks under certain conditions which we identify.
- In section 14.4, we present an implementation of discrete signals that includes a merge operation which combines simultaneously occurring values.
- After showing a way to memoize the values of a discrete signal in section 14.5, we show that this approach destroys the prerequisites for feedbacks. In section 14.6, we introduce lax arrows. Lax arrows provide a way to reorder parts of an arrow value in order to remove divergence for certain arrow feedbacks. We use lax arrows to make feedbacks possible in the presence of memoization.

Finally, we discuss related work in section 14.7, and give a conclusion and an outlook on further work in section 14.8 as well as acknowledgments in section 14.9.

All code presented in this paper is written in Haskell. Our concepts have been implemented in the Grapefruit library [9] and the *lax* package [10].

## 14.2 A SURVEY OF OUR DATA FLOW SYSTEM

Discrete signals can describe sequences of events, each of them occurring at a specific time and being parametrized by a certain value (for example, a key code in the case of key press events). They can also be composed to form new signals. A discrete signal is a value of type *DSignal val* for some type *val*. The internal structure of discrete signals is designed such that it aids a push-based execution of reactive systems. It cannot be accessed directly by the application programmer. However, we define the *meaning* of a signal to be a Haskell value which directly reflects the intuition behind the signal.

We define times to be non-negative real numbers whereas zero is the time where the reactive system starts. The type *Time* covers all strictly positive reals.<sup>2</sup> If  $dSignal :: DSignal\ val$  for some type *val* then  $\llbracket dSignal \rrbracket :: [(Time, val)]$  is the meaning of *dSignal*. The elements of the meaning are called occurrences. We say that the value *val* occurs at time *time* in the signal *dSignal* if  $(time, val) \in \llbracket dSignal \rrbracket$ . The fact that 0 is not a value of *Time* makes it impossible for a value to occur at the start of the reactive system.

The meaning of each signal *dSignal* fulfills the condition that for every *time* :: *Time*, the expression

$$all (>time) (tail (dropWhile (<time) (map fst \llbracket dSignal \rrbracket))))$$

is equivalent to *True*. Note that this restriction implies the following:

- The times of the occurrences are strictly ascending.
- For every time *time* there are only finitely many occurrences with a time less than *time*.
- $\perp$  can only occur as part of the value components.

The second and third point hold because otherwise the above expression would be equivalent to  $\perp$  instead of *True*.

Interaction with the environment is described by *circuits*. A circuit has an input and an output, both of which are typically tuples of discrete signals.<sup>3</sup> A circuit may produce occurrences in its output and change the outside world in response to occurrences in its input and external events. A circuit with input type *i* and output type *o* is a value of type *Circuit i o*. *Circuit* is an instance of *Arrow* and *ArrowLoop* [11] which allows complex circuits to be composed out of simpler ones. *Arrow* syntax [11] is typically used for this.

In the following, we show signals and circuits in action using a simple GUI application. During our walk through this example, we introduce several operations on signals and state their semantics. Our example application displays the current time in the format *HH:MM*. Beside the displayed time are two buttons for modifying the time, an hour incrementing button to the left and a minute incrementing button to the right. The source code of the application is shown in figure 14.1.

*clockApp* is a circuit based on the predefined circuit constructing functions *minutePulse*, *button* and *label* whose type signatures are given in figure 14.2. When *clockApp* is run, an *instance* of this circuit is created. This consists of instances of its subcircuits. A *minutePulse* instance outputs a signal in which the value  $()$  occurs every minute. An instance of *button initCap* denotes a button which will show up as part of a GUI. This button initially has the caption *initCap*. Everytime a value *newCap* occurs in the input, the button's caption changes to *newCap*. The value  $()$  occurs in the output whenever the user presses the button.

<sup>2</sup>We disregard the fact that the type of real numbers is actually not implementable.

<sup>3</sup>Hereby we consider  $()$  to be the 0-tuple and identify discrete signals with their corresponding 1-tuples.

```

clockApp :: Circuit () ()
clockApp = proc () → do
  rec let
    hPulse = filter (λm → m `mod` 60 ≡ 0) (count mPulse)
    update = merge (count (hPulse `merge` hPress))
                  (count (mPulse `merge` mPress))
    timeUpd = scan nextTime (0,0) update
    mPulse ← minutePulse → ()
    hPress ← button "H" → empty
    () ← label "00:00" → fmap timeStr timeUpd
    mPress ← button "M" → empty
  returnA → ()

count :: DSignal val → DSignal Int
count = scan (λnum _ → succ num) 0

nextTime :: (Int, Int) → MergeVal Int Int → (Int, Int)
nextTime (_, oldM) (First newH) = (newH, oldM)
nextTime (oldH, _) (Second newM) = (oldH, newM)
nextTime (_, _) (Both newH newM) = (newH, newM)

timeStr :: (Int, Int) → String
timeStr (h, m) = twoDigits (h `mod` 24) ++ " : " ++ twoDigits (m `mod` 60)

twoDigits :: Int → String
twoDigits n = reverse (take 2 (reverse ('0' : show n)))

```

FIGURE 14.1. Source code of the clock application

An instance of *label initText* denotes a text label. Setting the label's text is handled analogously to setting a button's caption. The label in our example is used to display the time.

The order of button and label circuits in the application circuit determines the order of the buttons and labels on the screen. Since the time shown by the label depends on the minute incrementing button, we have to use feedback as provided by *ArrowLoop*. This is done using **rec**. As a side effect, we are able to put all local variable definitions into a single **let** block at the beginning.

The example code uses several operations on discrete signals whose type declarations and semantics are shown in figure 14.3. Because it is impossible in

```

minutePulse :: Circuit () (DSignal ())
button      :: String → Circuit (DSignal String) (DSignal ())
label       :: String → Circuit (DSignal String) ()

```

FIGURE 14.2. Type signatures of circuit constructing functions

```

empty :: DSignal val
[[empty]]time = []

filter :: (val → Bool) → DSignal val → DSignal val
[[filter prd dSignal]]time = Prelude.filter (prd ∘ snd) [[dSignal]]time

fmap :: (val → val') → DSignal val → DSignal val'
[[fmap fun dSignal]]time = onVals (fmap fun) [[dSignal]]time

scan :: (val' → val → val') → val' → DSignal val → DSignal val'
[[scan acc init dSignal]]time = onVals (tail ∘ scanl acc init) [[dSignal]]time

onVals :: ([val] → [val']) → [(Time, val)] → [(Time, val')]
onVals valsFun occs = zip (map fst occs) (valsFun (map snd occs))

data MergeVal val1 val2 = First val1 | Second val2 | Both val1 val2

merge :: DSignal val1 → DSignal val2 → DSignal (MergeVal val1 val2)
[[merge dSignal1 dSignal2]]time = occsMerge [[dSignal1]]time [[dSignal2]]time

occsMerge :: [(Time, val1)] → [(Time, val2)] → [(Time, MergeVal val1 val2)]
occsMerge [] occs2 = occs2
occsMerge occs1 [] = occs1
occsMerge occs1 occs2 = let
    (headTime1, headVal1) = head occs1
    (headTime2, headVal2) = head occs2
in case compare headTime1 headTime2 of
    LT → (headTime1, First headVal1):
        occsMerge (tail occs1) occs2
    EQ → (headTime1, Both headVal1 headVal2):
        occsMerge (tail occs1) (tail occs2)
    GT → (headTime2, Second headVal2):
        occsMerge occs1 (tail occs2)

```

FIGURE 14.3. Type signatures and semantics of basic discrete signal functions

general to state the meaning of a *filter* result as a Haskell expression, we define meanings indirectly through *bound meanings*. For any  $dSignal :: DSignal$  and  $time :: Time$ , the bound meaning  $[[dSignal]]_{time}$  is equivalent to  $takeWhile ((\leq time) \circ fst) [[dSignal]]$ .

*empty* denotes a signal without any occurrences. It is used to state that the button captions never change. *fmap* applies a function to all occurring values. The helper function *count* counts the occurrences in a given signal. It uses the *scan* function which accumulates occurrence values and has similarities with the *scanl* function from the prelude. *scan* is also used to generate a signal of times from the *update* signal which describes time changes. *filter*<sup>4</sup> drops occurrences whose

<sup>4</sup>This function is different from the prelude function of the same name and can be distinguished from the prelude function because it belongs to a different module.

values do not fulfill a given predicate. This is used in the definition of *hPulse* to get a signal which has an occurrence every hour.

The most important function is *merge*. *merge* aggregates the occurrences of two signals. Thereby, it combines occurrences with equal time. Different external events are always considered to happen at different times. Therefore, the merges of pulse and press signals in the definition of *update* will not combine a pulse with a press. On the other hand, the outer merge of the *update* definition will combine each hour update with the corresponding minute update. If it would not, there would be two changes of the label on every full hour. One would change the hour digits and one the minute digits. Such behavior means a performance loss which is not critical in this case but becomes significant in situations where arbitrarily many signals are merged. Moreover, there would be an inconsistent intermediate state. Depending on the order of updates, there would be transitions like  $07:59 \rightarrow 08:59 \rightarrow 08:00$  or  $07:59 \rightarrow 07:00 \rightarrow 08:00$ . Such behavior can become crucial if signal occurrences do not just affect a display but control some safety-critical device. Note that our discrete signal semantics do not even allow multiple occurrences at the same time in the same signal.

Being a pure data flow system, our system lacks a switch function. Such a function would have type  $DSignal\ val \rightarrow DSignal\ (DSignal\ val) \rightarrow DSignal\ val$  and produce a signal which initially behaves like the first argument and switches to the behavior of any signal occurring in the second argument whenever a occurrence takes place there.

### 14.3 IMPLEMENTING CIRCUITS

The key idea of our push-based FRP implementation is that the task of actually running the system is left completely to event handlers. Initializing the system, including registering the event handlers, is all which has to be done in the first place. Control is then given to an event loop which lets the event handlers do their job. This leads us to an implementation of circuits where a circuit is just an I/O action which does the necessary initialization work:

```
newtype Circuit i o = Circuit (Kleisli IO i o) deriving (Arrow)
```

Note that we use GHC's generalized **newtype** deriving mechanism to carry the *Arrow* instance of *Kleisli IO* over to *Circuit*.

Since initialization done in the beginning might give rise to necessary finalization at the end, we allow the initialization action to output a finalization action in addition to the ordinary output. We use the *WriterArrow* transformer [12] for this:

```
newtype Circuit i o = Circuit (WriterArrow (IO :$ ()) (Kleisli IO) i o)
deriving (Arrow)
```

The `:$` type operator denotes type application and is taken from the *TypeCompose* package [5]. Using `IO :$ ()` instead of `IO ()` gives us a *Monoid* instance for free

where *empty* means no finalization and *mappend* means sequencing of finalizations. Thus, the finalization action of a circuit is automatically composed from the finalization actions of its parts.

The above implementation has a problem. It is not possible to provide a sensible *ArrowLoop* instance for *Circuit*. This can be demonstrated with the clock example. The input of the clock depends on the signal *mPress*. Event handler registration for the clock would need to know this signal in order to register a handler for reacting on presses of the *M* button. However, since this button would not have been constructed yet, this signal would not yet be known.

We solve this problem by only doing things like GUI widget creation in the first place and deferring event handler registration to the time when every other initialization has been done. Instead of writing a finalization action, the writer arrow now writes an action which registers the event handlers and returns the corresponding finalization action. This action is called a setup and has the type *IO:\$IO:\$()*. Again, we get a sensible *Monoid* instance. *empty* means no registration and no finalization while *mappend* denotes sequencing of registration and finalization actions. The definition of *Circuit* becomes the following:

```
newtype Circuit i o = Circuit (WriterArrow Setup (Kleisli IO) i o)
deriving (Arrow, ArrowLoop)

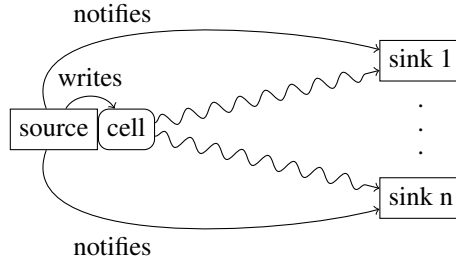
type Setup = IO:$IO:$()
```

With this solution, the clock builds a setup which depends on the yet unknown *mPress* signal which is no problem because of lazy evaluation. When the registration is performed, all subcircuits have been created and therefore all subcircuit outputs are known. In general, feedbacks work as long as I/O actions which use a circuit's input are not needed to compute the circuit's output. However, if they are needed, they cannot be put into the setup because the setup cannot contribute to a circuit's output. We will see an example of this situation in section 14.5 and show that the above implementation has a shortcoming here. Section 14.6 will show how to remove this shortcoming.

#### 14.4 IMPLEMENTING DISCRETE SIGNALS

Certain discrete signals describe sequences of external events whereby the occurrence values are the event parameters. For example, the output of a *minutePulse* instance describes a sequence of minute change events, and the output of a *button* instance describes the sequence of press events of the respective button. Event parameters are always *()* in these cases. A discrete signal which directly mirrors an event sequence is provided by a *discrete source*. Each instance of *minutePulse* and *button* covers a discrete source while *label* instances cover none. A signal *depends* on a source if it is the signal provided by this source or if it is formed from a signal which depends on this source by application of a signal function.

A *discrete sink* consumes a discrete signal and reacts on occurrences in it. Each *button* and *label* instance contains a discrete sink which changes the button's caption or the label's text, respectively.



**FIGURE 14.4.** Event handling

An occurrence in a discrete signal always originates from an external event. The reason is that all signal functions create only occurrences with times which have been taken from the occurrences of existing signals. Signal functions may transform and combine occurrence values, and drop occurrences but they never “invent” completely new occurrences. So a sink has only to react to external events which are made available through sources. The aim of the internal structure of *DSignal* values is to make the communication between sources and sinks possible.

Figure 14.4 shows what happens in response to an event. Each source has a mutable variable attached to it. This is called a *cell* and is created during circuit initialization. Normally, the cell contains  $\perp$ . Everytime an event occurs, the event’s parameter is stored in the cell. Afterwards, all sinks which might have to react to the event are notified of the event occurrence. They read the event parameter from the cell and determine whether the event occurrence leads to an occurrence in their input signal. If this is the case, they calculate the occurrence’s value and react to the occurrence. Finally, the cell’s content is reset to  $\perp$ .

We introduce a type *DSource* for discrete sources:

```
data DSource = DSource Unique Notifier
```

```
type Notifier = IO ()  $\rightarrow$  Setup
```

Each source consists of a unique identifier and a notifier. Source IDs are generated during circuit initialization via *newUnique*. There are *Eq* and *Ord* instances of *DSource* which define equivalence and ordering of sources in terms of equivalence and ordering of their IDs. These instances are needed since discrete sources are used as keys in maps, as we will see soon. Notifiers are functions which map event handlers to setups. The setup for a given event handler registers this handler at the source so that it gets called on every event of this source.

The central part of a signal’s internal structure is the *generator map*. This maps all sources, the signal depends on, to *generators*. A generator is an I/O action which outputs *Nothing* when an event of the corresponding source does not lead to an occurrence in the signal, and *Just val* if it leads to an occurrence of value *val*. The generator reads the source’s cell in order to determine the generator’s result. Since the *scan* function allows occurrence values to depend on the signal’s

history, a generator may receive references to mutable variables as its input. These mutable variables store values accumulated by *scan*. The references pointing to them are called *locals* and are provided to the generator in form of a nested tuple. The type definitions for generator maps and generators are as follows:

```
type GenMap locals val = Map DSource (Gen locals val)
type Gen locals val = locals → IO (Maybe val)
```

A discrete signal consists of an *I/O* action which creates any mutable variables for holding accumulated values, and of a generator map:

```
data DSignal val = ∀locals.DSignal (IO locals) (GenMap locals val)
```

The action for creating mutable state is run once by each sink which consumes the signal. Its result is used together with the generators to form the event handlers which are registered at the respective sources afterwards.

Using this definition of *DSignal*, all functions introduced in section 14.2 can be implemented. We present the implementations of *filter* and *merge*. They are shown in figure 14.5. We use the fact that *Map* is a functor and that  $(\rightarrow)$  *locals* and *IO* are functors and monads for pushing function applications inside generators and generator maps via *fmap* and *liftM2*. *merge* transforms its arguments' generator maps into *prepared maps*. These already work with the result's *locals* and extract the part, they are interested in, via *fst* or *snd*, respectively. In addition, they apply *First* or *Second*, respectively, to the occurrence values of the argument signals. When forming the result signal's generator map, generators belonging to the same source are combined via *combGen*. This makes sure that simultaneous occurrences are combined.

## 14.5 MEMOIZATION

Consider two different sinks whose signals are derived from a common signal by signal function applications. The *locals* creation action of the common signal is contained by the *locals* creation actions of both sinks' signals. The analogue holds for the generators of the common signal. *Locals* creation of the common signal is therefore done for every sink instead of just once, resulting in an extra time and possibly space cost. A further time penalty arises from the common signal's generators being called for every sink.

To solve these problems, we introduce the *memo* circuit. *memo* has type *Circuit (DSignal val) (DSignal val)* and is denotationally equivalent to the identity arrow *pure id*. In contrast to *pure id*, the output signal can be used multiple times without imposing the time and space overhead discussed above.

Event handling in the presence of memoization is depicted in figure 14.6. An instance of *memo* for a signal of type *DSignal val* covers a mutable variable of type *Maybe (Maybe val)* which is called a *box*. Normally, the content of the box is *Nothing* which means that there is no information concerning occurrences in the signal. During event handling for any of the signal's sources, the content is

```

filter prd (DSignal newLocals genMap) = DSignal newLocals genMap' where
  genMap'          = (fmap ∘ fmap ∘ fmap) outputConv genMap
  outputConv Nothing = Nothing
  outputConv (Just val) = if prd val then Just val else Nothing

merge (DSignal newLocals1 genMap1)
      (DSignal newLocals2 genMap2) = DSignal newLocals' genMap' where
  newLocals'          = liftM2 (,) newLocals1 newLocals2
  genMap'             = unionWith combGen prepGenMap1 prepGenMap2
  combGen              = (liftM2 ∘ liftM2) combVal
  combVal maybe1 maybe2 = case catMaybes [maybe1, maybe2] of
    []                → Nothing
    [mergeVal]       → Just mergeVal
    [First val1, Second val2] → Just $
    Both val1 val2

  prepGenMap1        = fmap ((∘fst) ∘ (fmap ∘ fmap ∘ fmap)) First
    genMap1
  prepGenMap2        = fmap ((∘snd) ∘ (fmap ∘ fmap ∘ fmap)) Second
    genMap2

```

FIGURE 14.5. Implementation of the *filter* and the *merge* function

changed to *Just Nothing* if there is no occurrence in the signal, or to *Just (Just val)* if the value *val* occurs. Every generator of the *memo* instance's output checks the content of the box and changes it if the box still contains *Nothing*. It then uses the content of the box to produce its output.

After all respective sinks have reacted to an event occurrence, the content of the box has to be reset to *Nothing*. The *memo* instance is responsible for doing that. Therefore, it has to be notified after event handling is complete. We extend the *DSource* type with a second notifier. Handlers registered via this notifier are called after all handlers registered with the first notifier have been executed.

In order to change the content of the box, the generator of the output signal calls the respective generator of the input signal. The locals argument used by this call is created by the *memo* instance during initialization. So in order to produce the output signal, the locals creation action of the input signal has to be executed. This means that this execution cannot be deferred by putting it into the setup but has to be performed during the first initialization stage. So at this time, the locals creation action of the input has to be already known. This is problematic if the memoized signal depends on a source which is situated after the *memo* instance.

Signals which correspond to a certain source use *return ()* for locals creation, independently of any I/O outputs. Therefore, locals creation actions of signals can be calculated purely functionally and could therefore be known early enough. Nevertheless, the output of an I/O action is completely unknown until the action

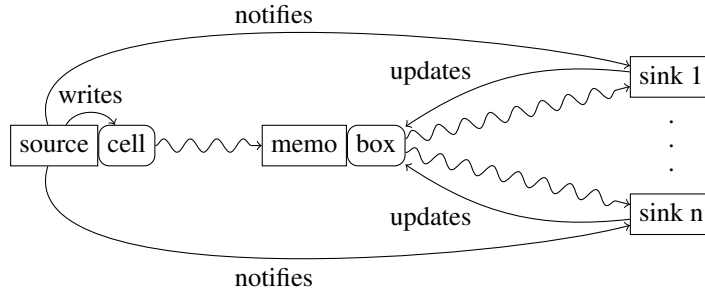


FIGURE 14.6. Event handling with memoization

has been executed completely. To illustrate this, take a look at the I/O Kleisli arrow

```
loop (first get >>> pure (uncurry (++) >>> put >>> pure (const ((, " . "))))
```

where  $get = Kleisli \$ const getLine$  and  $put = Kleisli \$ putStrLn$ . Only after the last *pure* arrow, it is known that the output of *loop*'s argument is  $((, " . ")$ . So *put* will work with a feedback of  $\perp$ , not  $" . "$ , and therefore fail.

So it is impossible to memoize a signal if this signal is derived from a signal which is fed back via *loop*. We will solve this problem in the next section. The idea is to automatically move all pure computations to the beginning of the initialization action. We develop a general arrow transformer to achieve this.

### 14.6 GETTING LAZY VIA RELAXING

We introduce a type *LaxArrow* of kind  $(* \rightarrow * \rightarrow *) \rightarrow (* \rightarrow * \rightarrow *)$ . For every *Arrow* instance *base*, *LaxArrow base* is an instance of *Arrow* and *ArrowLoop*. In addition, there exist the following functions:

$$\begin{aligned} impure &:: (ArrowLoop base) \Rightarrow base \ i \ o && \rightarrow LaxArrow \ base \ i \ o \\ runLax &:: (Arrow base) && \Rightarrow LaxArrow \ base \ i \ o \rightarrow base \ i \ o \end{aligned}$$

Applying *impure* to a base arrow value results in a so-called *impure particle*. The *Arrow* and *ArrowLoop* methods of *LaxArrow base* can be used to build further lax arrow values from impure particles. Note that these methods add only pure computations.

Now, the idea is to perform all these pure computations at the beginning. *runLax* achieves this by generating a base arrow value starting with a pure arrow value, called the *converter*. The converter receives the outputs of all impure particles and the input of the complete arrow value as its input. It produces the inputs of the impure particles as well as the output of the whole arrow value. Following the converter there are the base arrow values which were used to construct

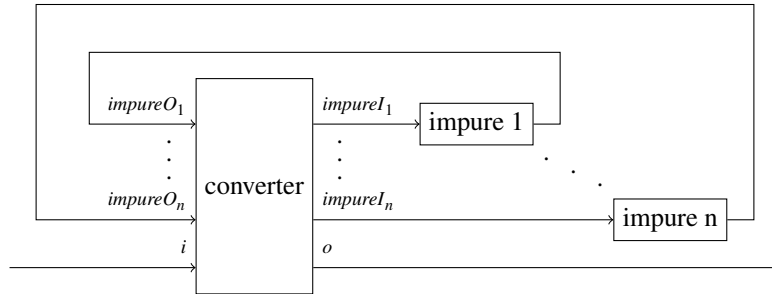


FIGURE 14.7. Structure of *runLax* results

the impure particles. Their results are immediately fed back into the converter. Figure 14.7 shows the structure of *runLax* results graphically.

The feedback example from the last section can now be made working by replacing *get* and *put* with *impure get* and *impure put*, respectively, and applying *runLax* to the whole expression. The result is equivalent to

$$\text{loop} (\text{loop} (\text{pure conv} \gg \text{second get}) \gg \text{second put})$$

where *conv* is defined as follows:<sup>5</sup>

$$\text{conv} \sim (\sim(i, \text{putO}), \text{getO}) = ((((), \text{getO} ++ " . "), i)$$

The ". ." now appears directly in the input of *put* as generated by the converter. This is because ". ." is fed back inside the converter. This feedback is not visible in the above definition of *conv* because this definition is already in a simplified form. Further note that *put* does not receive the result of *get* directly but only via feedback and through the converter. It is important that *get*'s output is fed back immediately after *get* because otherwise it would be available too late.

The type definition of *LaxArrow* is given in figure 14.8. Internally, a *LaxArrow* value consists of the converter function and the *base generator*. The base generator is a function which, given the converter as its argument, constructs the complete arrow value yielded by *runLax*. However, the base generator can also be applied to non-pure arrow values and to arrow values which use different types in place of the input and output types. This generalization is necessary for the implementation of ( $\gg$ ).

Using this definition, the implementation of *runLax* is straightforward. The definitions of the other lax arrow functions are quite complicated in part because of necessary tuple restructuring. We only give the general ideas here, for the exact definitions, the reader is referred to the source code of the *lax* package [10]:

<sup>5</sup>Note that *conv* is not exactly the converter function because the input and output tuples of the converter have a different structure than the arguments and results of *conv*. We have chosen this definition of *conv* to make the presentation a bit simpler.

```

data LaxArrow base i o =  $\forall$  impureI impureO.
    LaxArrow ((impureO, i)  $\rightarrow$  (impureI, o))
    (BaseGen base impureI impureO)
type BaseGen base impureI impureO =  $\forall$  i' o'.
    base (impureO, i') (impureI, o')  $\rightarrow$ 
    base i' o'

```

**FIGURE 14.8.** Implementation of the *LaxArrow* type

- Results of *impure* have a base generator which adds a single cycle containing the argument of *impure*. The converter function does only tuple restructuring.
- The converter function of a *pure* result applies *pure*'s argument to the input. The base generator does only tuple restructuring.
- The base generator of a ( $\gg$ ) result is basically the composition of the arguments' base generators. The converter function is responsible for feeding the output of the first arrow to the second arrow.
- *first* and *loop* do not touch the base generator. They basically apply *first* or *loop*, respectively, to the converter function.

We now obtain an improved implementation of *Circuit* by replacing *Kleisli IO* with *LaxArrow (Kleisli IO)*. Using this definition, *memo* instances are fully functional also in the presence of feedbacks.

## 14.7 RELATED WORK

The first push-based FRP implementation was developed for the Haskell GUI library *FranTk* [13, 14]. Alas, *FranTk*'s merge function does not detect simultaneous occurrences and therefore produces multiple occurrences instead of combined occurrences. Instead of a circuit arrow, *FranTk* uses a *GUI* monad which does not support feedbacks via a *MonadFix* instance. Feedbacks are implemented using so-called *wires* and *listeners* which leads to a less declarative style.

*FrTime* [3, 2] is a push-based FRP implementation in Scheme. *FrTime* runs an event loop in a separate thread. This makes it possible to incrementally develop and test reactive programs in the interactive *DrScheme* environment. There is a merge function for discrete signals which does not combine simultaneously occurring values. On the other hand, continuous signals in *FrTime* respect simultaneity. Feedbacks can be implemented declaratively using **letrec**.

*Frappé* [4] is an FRP library written in Java. It implements discrete and continuous signals as *JavaBeans*. Signals notify dependent signals about value occurrences and value changes by calling listener methods. This has the consequence that simultaneity cannot be detected as multiple occurrences at the same time are handled by independent method invocations. Feedbacks can be achieved only

imperatively. First, a partially-defined signal is constructed. The missing information is provided later when necessary signals are known.

*Fudgets* [1] is a Haskell GUI library. Its central concept is the *fudget* which is similar to a circuit with a single discrete input signal and a single discrete output signal. Fudgets are implemented as stream transformers using a continuation-passing style. While feedbacks are possible, simultaneously occurring values are not combined. Handling of events and modifying the environment needs time proportional to the depth of the respective fudget in the fudget hierarchy.

The Haskell library *Yampa* [8] is a popular example of a pull-based approach to FRP. *Yampa* represents discrete signals as continuous signals over *Maybe* types<sup>6</sup> where applications of *Just* represent occurrences while *Nothing* stands for the absence of occurrences. The downside of this approach is that *Yampa* cannot enforce that values occur at discrete times. Due to the pull-based implementation, the time for updating the system state grows linearly with the system size. On the other hand, *Yampa* supports recursive signal definitions which are not possible with our approach. Feedbacks and simultaneity are no problems in *Yampa*.

The declarative data flow languages *Signal* [6] and *Lustre* [7] are tailored to the concepts we dealt with in this paper. Like *Yampa*, they allow recursive signal definitions. The compiler even checks that such definitions do not lead to cyclic value dependencies. Furthermore, certain properties of occurrence times can be checked statically. Simultaneous occurrences are identified and handled correspondingly. In *Lustre*, time and space leaks are prevented and the adherence to given time and space constraints can be checked. Compilation of *Lustre* programs generates efficient code by using finite automata as an intermediate representation.

## 14.8 CONCLUSIONS AND FURTHER WORK

We have presented a push-based implementation of a data flow library. Our system allows for declarative descriptions of reactive systems, including system with feedbacks. To make feedbacks work, we employed a writer arrow for delaying parts of the system initialization. We developed lax arrows to enable feedbacks in combination with memoization. With lax arrows, parts of an arrow value are rearranged so that pure computations are performed first which makes their outputs available to all impure computations. Our implementation also provides a merge function for discrete signals which combines simultaneously occurring values instead of generating multiple occurrences.

Ongoing research deals with extending our signal implementation to support switching. Furthermore, we investigate implementation techniques for continuous signals. In addition, we are working on support for circuits whose structure changes over time, and for incrementally updating continuous signals. *merge* could execute generators and locals creation actions of its argument signals concurrently. It should be analyzed how this can help improving performance.

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<sup>6</sup>Actually, *Yampa* uses a type *Event* which is isomorphic to *Maybe*.

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