Modeling variability of renewable energy sources by a semi-Markov process for power generation prediction

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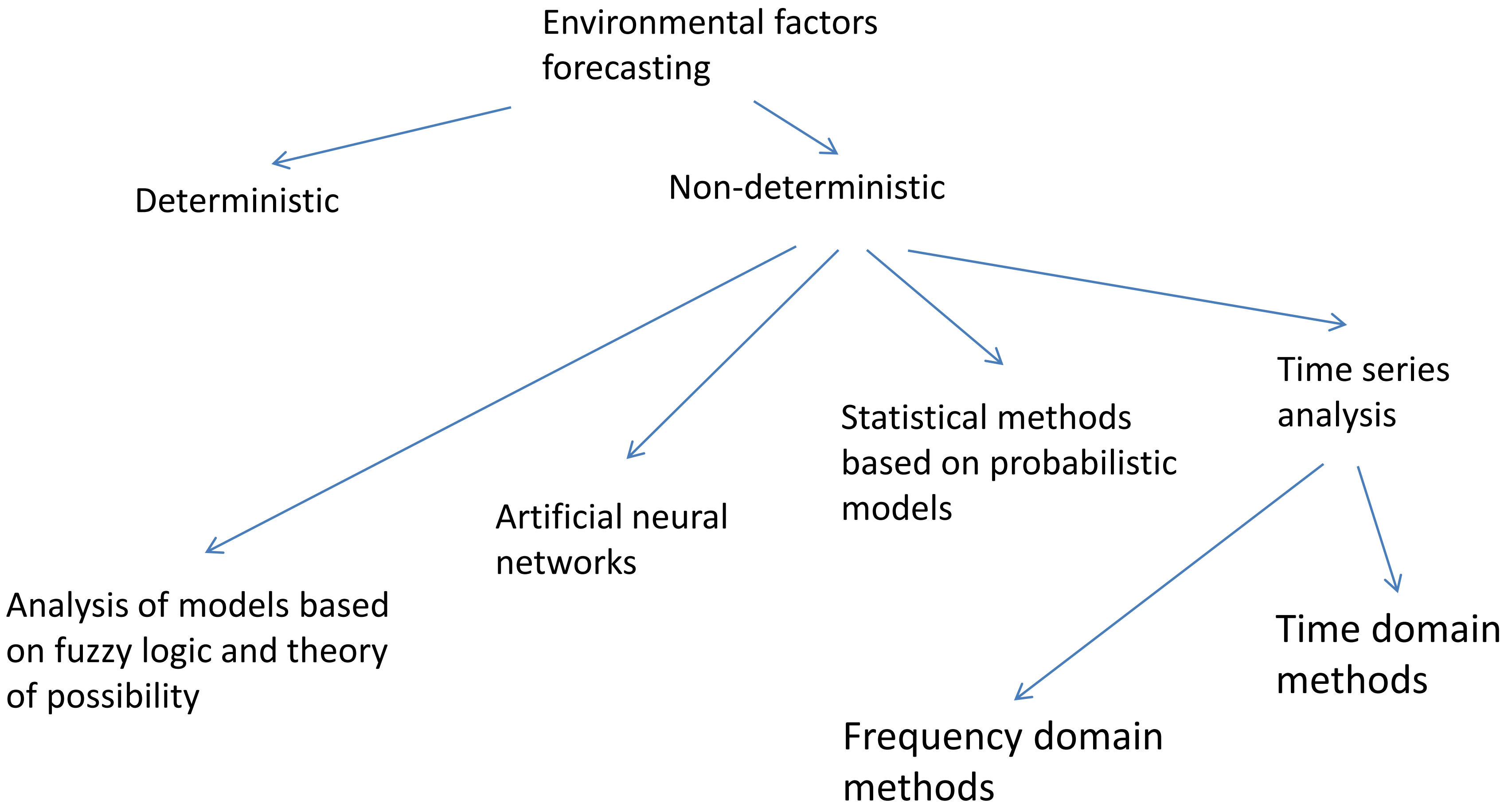
Overview of forecasting methodology



- Environmental factors forecasting
  - Deterministic
  - Non-deterministic
    - Analysis of models based on fuzzy logic and theory of possibility
    - Artificial neural networks
    - Statistical methods based on probabilistic models
    - Time series analysis
      - Frequency domain methods
      - Time domain methods

The proposed model's assumptions (1)

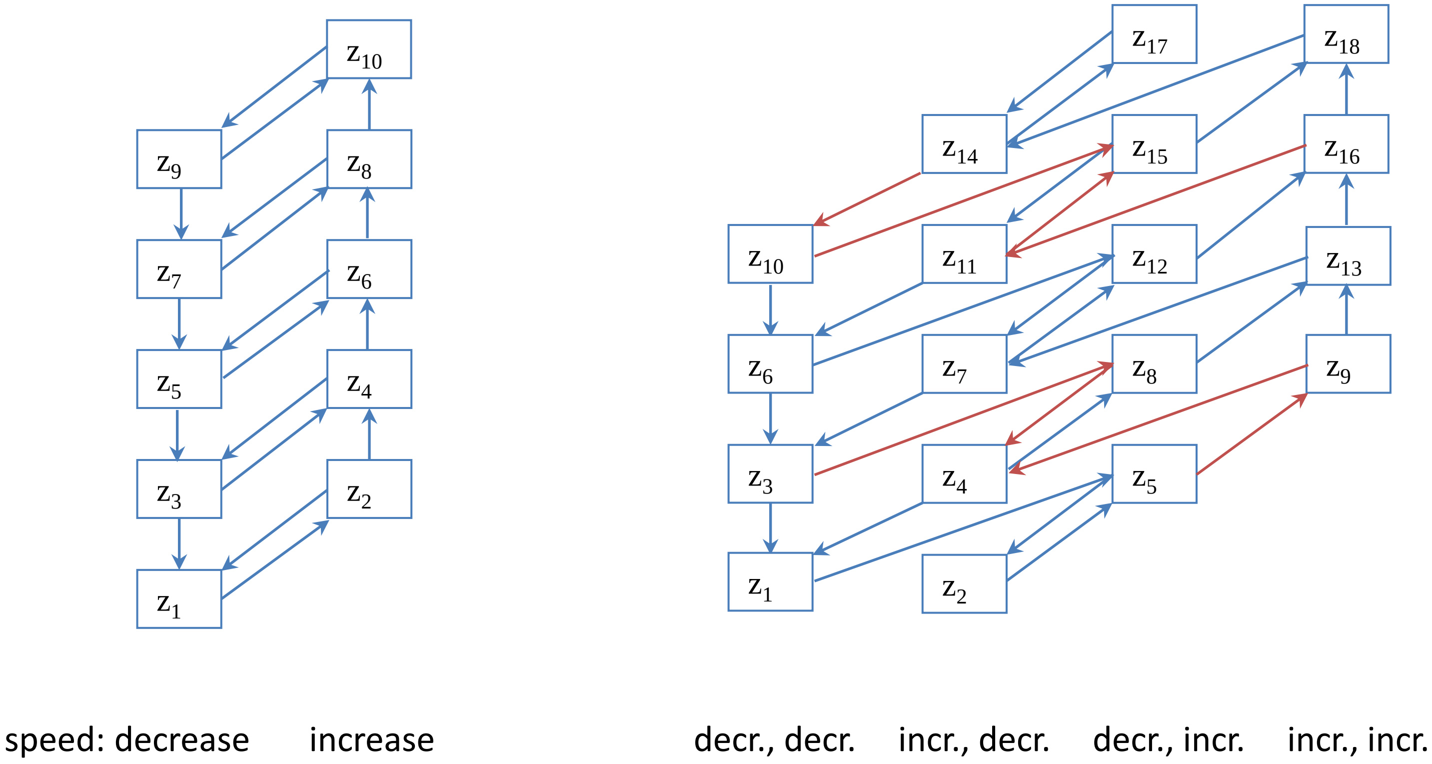
$\{I_1,\ldots,I_m\}$ – the set of disjoint intervals into which the range of possible wind sppeds is divided

$Z=\{z_1,\ldots,z_n\}$ – the state space of the (semi-Markov) stochastic process serving as the wind speed variability model

To each interval I_x , $x=1\ldots m$, there is assigned a number of states in Z. They correspond to various sequences of intervals in which the wind speed had stayed before it entered I_x

The number of previously entered intervals taken into account in constructing the state space Z depends on the model's degree of accuracy

The proposed model's assumptions (2)



The state space Z for the first and second degrees of accuracy, m=6

The proposed model's assumptions (3)

$Y = \{Y_t : t \geq 0\}$ – the semi-Markov process with the state space Z, modeling the wind speed variability

$X = \{X_k : k \geq 0\}$ – the embedded Markov chain of Y, i.e.
X_k - the state of Y at the moment of its k-th state change
X_0 - the state of Y at t = 0

$P = [p_{ij}]_{i,j=1\ldots n}$ – the transition matrix of X, i.e. $p_{ij} = \Pr(X_k = z_j \mid X_{k-1} = z_i)$, $p_{ii} = 0$, i,j = 1…n
P is identical for every k, i.e. X is homogenous
E.g. for the first degree of accuracy we have

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{21} & 0 & 0 & p_{24} & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{31} & 0 & 0 & p_{34} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{43} & 0 & 0 & p_{46} & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{53} & 0 & 0 & p_{56} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{65} & 0 & 0 & p_{68} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{75} & 0 & 0 & p_{78} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{87} & 0 & 0 & p_{8,10} \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{97} & 0 & 0 & p_{9,10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The proposed model's assumptions (4)

T_k – the moment of the k-th state change of Y, $k \geq 0$, $T_0 = 0$

S_{ij} – the time spent by Y in the state z_i provided that the next state is z_j

F_{ij} – the distribution function of S_{ij}, i.e.

$$F_{ij}(t) = \Pr\left(T_k - T_{k-1} \leq t \;\middle|\; X_k = z_j, X_{k-1} = z_i\right),\ k \geq 1$$

P and F_{ij} can be obtained from the statistical analysis of wind speed records

Possible applications

- Determining parameters which characterize the wind power production process – forecasting their future values

  Exemplary parameters:

  Total/average expected energy output during a given time period

  The probability that during a given period the wind speed (wind turbine's output power) will remain within certain limits

- Performing Monte Carlo simulation in order to determine values that are difficult to compute analytically

An example (1)

It will be demonstrated how to analytically determine the total expected energy output during a given time period

π_i - the power generated when Y is in the state z_i

$G_i(u,t)$ - the expected value of the energy produced in the time period [u,t], provided that **at the moment u the process Y enters** the state z_i.

$G_i(0,t)$, i=1,…,n satisfy the following set of equations

$$G_i(0,t) = \pi_i t \sum_{j \neq i} p_{ij}[1 - F_{ij}(t)] + \sum_{j \neq i} p_{ij} \int_0^t [\pi_i u + G_j(u,t)] dF_{ij}(u)$$

As Y is semi-Markov, it „forgets" its history at each state change, so that $G_i(s,t) = G_i(0,t - s)$, i=1…n, and the above equations transform to:

An example (2)

$$G_i(0,t) = \pi_i \sum_{j \neq i} p_{ij} \{ t[1 - F_{ij}(t)] + \int_0^t u dF_{ij}(u) \} + \sum_{j \neq i} p_{ij} \int_0^t G_j(0, t-u) dF_{ij}(u)$$

Let

$$H_{ij}(t) = t\left[1 - F_{ij}(t)\right] + \int_0^t u dF_{ij}(u) = E\left[\min\left(S_{ij}, t\right)\right]$$

hence we have:

(*) $$G_i(0,t) = \pi_i \textstyle\sum_{j \neq i} p_{ij} H_{ij}(t) + \sum_{j \neq i} p_{ij} \int_0^t G_j(0, t-u) dF_{ij}(u)$$

Let $\Gamma_i(s) = \mathcal{L}\{G_i(0,t)\}$, $\Phi_{ij}(s) = \mathcal{L}\left\{f_{ij}(t)\right\} = \mathcal{L}^*\left\{F_{ij}(t)\right\}$

It also holds that $\mathcal{L}\left\{H_{ij}(t)\right\} = [1 - \Phi_{ij}(s)]/s^2$

$\mathcal{L}$ and $\mathcal{L}^*$ denote the Laplace and Laplace-Stieltjes transforms

An example (3)

Applying $\mathcal{L}$ to both sides of (*) we obtain:

$$\Gamma_i(s) = \frac{\pi_i}{s^2}\sum_{j\neq i} p_{ij}[1-\Phi_{ij}(s)] + \sum_{j\neq i} p_{ij}\Phi_{ij}(s)\Gamma_j(s)$$

which can be written in a condensed matrix form as follows:

(**) $$A(s)\begin{bmatrix}\Gamma_1(s)\\ \vdots \\ \Gamma_n(s)\end{bmatrix} = \begin{bmatrix}\frac{\pi_1}{s^2}\sum_{j\neq 1} p_{1j}[1-\Phi_{1j}(s)]\\ \vdots \\ \frac{\pi_n}{s^2}\sum_{j\neq n} p_{nj}[1-\Phi_{nj}(s)]\end{bmatrix}$$

where the elements of the matrix A(s) are given by:

$$a_{ij}(s) = \delta_{ij} - p_{ij}\Phi_{ij}(s) \quad i,j = 1 \dots n$$

and δ_{ij} is the Kronecker's delta.

An example (4)

For the first degree of accuracy the matrix A(s) has the following form

$$\begin{bmatrix}
1 & -\Phi_{12}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-p_{21}\Phi_{21}(s) & 1 & 0 & -p_{24}\Phi_{24}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
-p_{31}\Phi_{31}(s) & 0 & 1 & -p_{34}\Phi_{34}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -p_{43}\Phi_{43}(s) & 1 & 0 & -p_{46}\Phi_{46}(s) & 0 & 0 & 0 & 0 \\
0 & 0 & -p_{53}\Phi_{53}(s) & 0 & 1 & -p_{56}\Phi_{56}(s) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -p_{65}\Phi_{65}(s) & 1 & 0 & -p_{68}\Phi_{68}(s) & 0 & 0 \\
0 & 0 & 0 & 0 & -p_{75}\Phi_{75}(s) & 0 & 1 & -p_{78}\Phi_{78}(s) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -p_{87}\Phi_{87}(s) & 1 & 0 & -p_{8,10}\Phi_{8,10}(s) \\
0 & 0 & 0 & 0 & 0 & 0 & -p_{97}\Phi_{97}(s) & 0 & 1 & -p_{9,10}\Phi_{9,10}(s) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Phi_{10,9}(s) & 1
\end{bmatrix}$$

For higher degrees of accuracy A(s) has more non-zero elements, however it remains sparse.

Notwithstanding the sparsity of A(s), it is practically impossible to find a closed-form solution of (**) for other than small values of n. Such a solution would express $\Gamma_i(s)$ in terms of p_{ij} and $\Phi_{ij}(s)$, i,j=1…n. Therefore, (**) has to be solved by means of an approximate numerical method.

The algorithm to compute total expected energy output in a given time period

Let $\{s_1, s_2, \ldots, s_k\}$ be a set of complex numbers that discretizes the vertical line in the complex space, used to find $G_i(0,t)$ as the inverse of $\Gamma_i(s)$, i=1…n, according to:

$$(***) \qquad G_i(0,t) = \frac{1}{2\pi i} \lim_{y \to \infty} \int_{x-iy}^{x+iy} e^{st} \Gamma_i(s) ds$$

<u>The algorithm</u>

```
for h=1 to k do {
   for i=1 to n do {
      find $\Gamma_i(s_h)$ by solving (**) for $s=s_h$ ## see the remark
   }
   for i=1 to n do {
      update $G_i(0,t)$ as given by (***) for $s=s_h$
   }
}
```

Remark: It is only necessary to store $\Gamma_i(s_h)$, i=1…n, for the current h

Computation of other parameters related to wind speed variability

Other parameters, e.g. the probability that during a given time period the wind speed will remain within certain limits, or the expected number of times it will cross these limits, can be foud in a similar way.

E.g., let $N_i(u,t)$ be the expected number of times, in the [u,t] period, the wind speed crosses the lower bound of I_a from above, or the upper bound of I_b from below, $a \leq b$, provided that **at the moment u the process Y enters** the state z_i.
The basic set of equations, analogous to that in slide 8, is the following:

$$N_i(0,t) = \sum_{j \neq i} p_{ij} \int_0^t \left[\gamma(i,j) + N_j(u,t)\right] dF_{ij}(u)\,, \qquad i = 1 \dots n$$

where

$\gamma(i,j) = 1$ for $i \in Z_1$, $j \in Z_2$; otherwise $\gamma(i,j) = 0$
$Z_1 = \{z \in Z\colon a \leq x(z) \leq b\}$
$Z_2 = Z \setminus Z_1$

The solution method is similar to that presented on slides 8-12.

Thank you for your attention